End Semester Examination

Physics IV,
B. Math.,
January - May 2021.
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May 10^{th}, 2021, Morning Session.
Duration: 3 hours.
Total points: 80

Please give arguments where necessary. If it is unclear from your answer why a particular step is being taken, full credit will not be awarded. Grades will be awarded not only based on what final answer you get, but also on the intermediate steps.

1. A particle, moving in an infinite square well of width L, has the following initial wave function:

$$\psi(x,0) = A\sin^3\left(\frac{\pi x}{L}\right)$$

Find:

   (a) The constant A. [3]
   (b) The time-dependent wave-function $\psi(x,t)$. [4]
   (c) The expectation value of energy E. [3]

2. An operator $\hat{A}$, representing observable A, has two eigenfunctions ψ_1 and ψ_2, with eigenvalues a_1 and a_2 respectively. Similarly, another operator $\hat{B}$, representing observable B, has eigenfunctions ϕ_1 and ϕ_2, with eigenvalues b_1 and b_2, respectively. The eigenfunctions are related to each other by the following two relations:

$$\begin{aligned}\psi_1 &= \frac{1}{5}(3\phi_1 + 4\phi_2)\\ \psi_2 &= \frac{1}{5}(4\phi_1 - 3\phi_2)\end{aligned}$$

   (a) Observable A is measured and the result a_1 is obtained. What is the wave-function of the system immediately after this measurement? [1]
   (b) If the observable B is measured immediately after the first measurement, what are the probabilities of obtaining b_1 and b_2 respectively as a result of this measurement (of B)? [2+2]

(c) Suppose the result of the second measurement was b_2. Immediately A is measured. what are the probabilities of obtaining a_1 and a_2 respectively as a result of this measurement (of B)? [2.5+2.5]

All your answers must be accompanied by a short explanation.

3. Consider the double delta-function well potential, which has a delta-function well at two positions, $x = a$ and $x = -a$. The potential is given by

$$V(x) = -\alpha \left[\delta\left(x+a\right) + \delta\left(x-a\right)\right] \tag{1}$$

   (a) Show that there is always one even bound state. [5]

   (b) Show that there is an odd bound state only if α is larger than a constant determined by the other parameters $(\hbar, m, a)$ entering the Schrödinger Equation. Find the constant. [5]

4. A quantum simple harmonic oscillator (with mass m and frequency ω) is known to be in a quantum state such that a measurement of its energy will give $\frac{1}{2}\hbar\omega$ and $\frac{3}{2}\hbar\omega$ with equal probability.

   (a) Write down its wave-function as a function of time (It is okay if you keep it in terms of the operators $\hat{a}_+$ and $\hat{a}_-$ and the ground state wave-function of the quantum SHO). [4]

   (b) Calculate the expectation values of the operators $\hat{x}$, and $\hat{p}$ as a function of time. [3]+[3]

5. A train with proper length L moves with a speed of $\frac{5c}{13}$ with respect to the ground. A ball is thrown from the back-end of the train towards the front with a speed of $\frac{c}{3}$ with respect to the train. From the view of someone on the ground,

   (a) How much time the ball spend in the air before hitting the front-end. [5]

   (b) How much distance does it move in that time? [5]

6. A train moves towards a tunnel with speed v. Both the train and the tunnel have proper length L. Its front-end (i.e., the end closer to the tunnel) contains a bomb, which is designed to explode when the front-end of the train coincides with the far-end of the tunnel. When the back-end of the train coincides the near-end of the tunnel, a sensor at the back of the train tells the bomb to disarm (i.e., not to explode). The signal from the sensor must reach the bomb for the de-activation to happen. The sensor can send out signals at speed of light. Explain whether the explosion happens or not from the viewpoint of both the (i) train-frame (ii) tunnel-frame. [3+12].

7. (a) Two identical particles of rest-mass m, moving in opposite directions with equal speed βc, collide and coalesce into a single particle of rest-mass M, along with emitting a photon (particle of light). Argue that the resultant particle of rest-mass M cannot be stationery. [5]

(b) What are the energies of the particle with rest-mass M and the photon? [2.5+2.5]

(c) What are the magnitudes of the relativistic momenta of M and the photon? [2.5+2.5]